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Structure of tilted vortices and angular dependence of the lower critical field in anisotropic (d + s)-wave superconductors

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Abstract

The structure of a single tilted flux line in an anisotropic (d + s)-wave superconductor has been analysed within the Ginzburg–Landau theory generalized for two complex order parameter components. The angular dependence of the lower critical field H_{c1} and the vortex structure have been studied for different orientations of an applied magnetic field with respect to the crystal axes. In the limit of weak anisotropy (which can be treated within the Ginzburg–Landau theory with an anisotropic mass tensor), the arrangement of s-wave unit vortices induced in the core is shown to strongly depend on the field orientation. For a large-anisotropy limit (i.e. for layered Josephson coupled systems) the fourfold anisotropy of H_{c1} is shown to be dominated by the contributions which come from the Josephson string regions.

1. Introduction

In recent years, the mixed state of high-T_c superconductors has attracted a great deal of interest. In particular, this interest is stimulated by experiments which provide good evidence for an order parameter (OP) with mixed symmetry (i.e., dominant d wave and subdominant s wave) in these compounds. A great number of features of such materials can be treated within the Ginzburg–Landau (GL) theory with an OP consisting of two components corresponding to the d- and s-wave pairing. It is obvious that the presence of two OP components influences the structure of vortices. As shown in a number of works (see e.g. references [1–4]), a flux line in such materials contains several unit vortices with different OP components. Since high-T_c cuprates are layered structures, the concrete form of the extended GL theory depends on the ratio s/ξ_c (s is the interlayer distance, $\xi_c(T)$ is the coherence length along the c-axis):

(i) When $s/\xi_c \ll 1$ (the weak-anisotropy limit), we have anisotropic 3D superconductivity. This situation can be described within the framework of GL theory with an anisotropic mass tensor.

(ii) Quasi-two-dimensional (2D) superconductivity (the strong-anisotropy limit: $s/\xi_c \gg 1$). In this case the system consists of superconducting layers coupled by Josephson interaction. Such compounds are described by the Lawrence–Doniach (LD) functional of free energy.

Previously, for the case of a weak anisotropy, the vortex structure in (d + s)-wave superconductors has been studied in detail only for the magnetic field direction parallel to the c-axis (in this special case the structure of the flux line does not depend on the value of s/ξ_c. As shown in a number of works [1–4, 7], the structure of singular flux lines in (d + s)-wave superconductors exhibits a fourfold symmetry in contrast to the case for nonsingular vortices [8]. The core structure for an arbitrary magnetic field orientation has been investigated numerically by Vicente Alvarez *et al* [9]. It was shown that as the magnetic field tilts from the c-axis, the fourfold symmetry of a flux line turns into a twofold one. As the angle between the magnetic field direction and the c-axis increases, the two s-wave unit vortices move off the centre of the flux line, while the other two s-wave vortices move towards the centre. Previously, a similar change of vortex structures in a tilted magnetic field has been studied analytically for heavy-fermion systems [10]. It should be noted that the vortices with twofold symmetry have been studied [11] for the magnetic field orientation along the c-axis in the case where the subdominant s-wave OP component is nonzero far from the vortex.

In the case of a strong anisotropy, the flux line is a stack of 2D pancakes in superconducting layers, which are connected by Josephson strings (see e.g. papers [5, 6]). As shown for the conventional one-component layered superconductors, the structure of such a flux line strongly depends on the magnetic field direction (see the detailed analysis in e.g. reference [5]). When the magnetic field is close to the c-axis, the picture is very similar to that of the three-dimensional (3D) theory. Deviations from the 3D theory are most substantial for field orientations close to the layers.

In this paper we develop an analytical description of a single vortex structure in a ($d_{x^2-y^2}$ + s)-wave cuprate for an arbitrary direction of the external magnetic field with respect to the crystalline axes for both weak- and strong-anisotropy limits. We restrict ourselves to the case of a pure d-wave OP far from the flux line. We find that the arrangement of s-wave vortices crucially depends on the angle between the magnetic field direction and the c-axis. Taking into account the distribution of a subdominant s-wave OP component in the core of a tilted vortex, we calculate the angular dependence of the lower critical magnetic field H_{c1}.

2. Tilted vortex structure in anisotropic 3D superconductors (the weak-anisotropy limit, $s \ll \xi_c$)

We start with the case of a ($d_{x^2-y^2}$ + s)-wave superconductor with an anisotropic mass tensor. Let us consider the Ginzburg–Landau functional [3, 9] which is generalized for the case of two components of the OP Ψ_d and Ψ_s corresponding to the $d_{x^2-y^2}$-wave and s-wave pairing, respectively:

$$F = \int \Big\{ [a_\mathrm{d}|\Psi_\mathrm{d}|^2 + a_\mathrm{s}|\Psi_\mathrm{s}|^2 + \frac{b_\mathrm{d}}{2}|\Psi_\mathrm{d}|^4 + \frac{b_\mathrm{s}}{2}|\Psi_\mathrm{s}|^4 + \beta_1|\Psi_\mathrm{d}|^2|\Psi_\mathrm{s}|^2 + \frac{\beta_2}{2}(\Psi_\mathrm{d}^2\Psi_\mathrm{s}^{*2} + \Psi_\mathrm{d}^{*2}\Psi_\mathrm{s}^2)$$
$$+ K_\mathrm{d}|(\boldsymbol{\Pi}_\parallel + \gamma_\mathrm{d}\Pi_z \boldsymbol{z}_0)\Psi_\mathrm{d}|^2 + K_\mathrm{s}|(\boldsymbol{\Pi}_\parallel + \gamma_\mathrm{s}\Pi_z \boldsymbol{z}_0)\Psi_\mathrm{s}|^2$$
$$+ K_\mathrm{sd}[((\Pi_x\Psi_\mathrm{s})^*\Pi_x\Psi_\mathrm{d} - (\Pi_y\Psi_\mathrm{s})^*\Pi_y\Psi_\mathrm{d}) + \mathrm{c.c.}] \Big\} \,\mathrm{d}V \qquad (1)$$

where

$$\boldsymbol{\Pi}_\parallel = \boldsymbol{\nabla}_{\boldsymbol{r}} - \mathrm{i}\frac{2\pi}{\Phi_0}\boldsymbol{A}_\parallel \qquad \Pi_z = \nabla_z - \mathrm{i}\frac{2\pi}{\Phi_0}A_z \qquad \boldsymbol{r} = (x, y)$$

(Φ_0 is the flux quantum), $\boldsymbol{A} = (\boldsymbol{A}_\parallel, A_z)$, $\boldsymbol{H} = \mathrm{curl}\,\boldsymbol{A}$, $\Psi_\mathrm{d}(\boldsymbol{r})$ and $\Psi_\mathrm{s}(\boldsymbol{r})$ are the components of the OP, $a_\mathrm{s} = \alpha_\mathrm{s}(T - T_\mathrm{cs})$, $a_\mathrm{d} = \alpha_\mathrm{d}(T - T_\mathrm{cd})$, and γ_s, γ_d are the anisotropy parameters for the s- and d-wave OP components. We choose x, y, z lying along the crystallographic a-, b-, and c-axes, respectively.

The equations for the order parameters are obtained by varying the free energy with respect to Ψ_d^* and Ψ_s^*:

$$\begin{aligned} &a_\mathrm{d}\Psi_\mathrm{d} + b_\mathrm{d}|\Psi_\mathrm{d}|^2\Psi_\mathrm{d} + \beta_1|\Psi_\mathrm{s}|^2\Psi_\mathrm{d} + \beta_2\Psi_\mathrm{s}^2\Psi_\mathrm{d}^* - K_\mathrm{d}\boldsymbol{\Pi}^2\Psi_\mathrm{d} - K_\mathrm{sd}[\Pi_x^2 - \Pi_y^2]\Psi_\mathrm{s} = 0 \\ &a_\mathrm{s}\Psi_\mathrm{s} + b_\mathrm{s}|\Psi_\mathrm{s}|^2\Psi_\mathrm{s} + \beta_1|\Psi_\mathrm{d}|^2\Psi_\mathrm{s} + \beta_2\Psi_\mathrm{d}^2\Psi_\mathrm{s}^* - K_\mathrm{s}\boldsymbol{\Pi}^2\Psi_\mathrm{s} - K_\mathrm{sd}[\Pi_x^2 - \Pi_y^2]\Psi_\mathrm{d} = 0. \end{aligned} \tag{2}$$

Let us introduce the unit vector $\boldsymbol{n}$ along a vortex line: $\boldsymbol{n} = (\sin\theta\cos\alpha, \sin\theta\sin\alpha, \cos\theta)$. We assume $T_\mathrm{cs} < T_\mathrm{cd}$, and T is close to T_cd ($|a_\mathrm{s}| \ll |a_\mathrm{d}|$). In this case the dominant d-wave OP component induces a small subdominant s-wave component in a flux line which can be obtained using perturbation theory with a small parameter $a_\mathrm{s}/|a_\mathrm{d}|$. Hereafter we consider an extreme type-II limit, where the coupling to the vector potential can be ignored while studying the core structure of an isolated vortex line. For the sake of convenience, we rescale and rotate the coordinate system so that the new one is isotropic and the vortex line lies along a new z-axis:

$$\begin{aligned} \tilde{x} &= [x\cos\alpha + y\sin\alpha]\cos\tilde{\theta} - \gamma_\mathrm{d} z\sin\tilde{\theta} \\ \tilde{y} &= -x\sin\alpha + y\cos\alpha \\ \tilde{z} &= [x\cos\alpha + y\sin\alpha]\sin\tilde{\theta} + \gamma_\mathrm{d} z\cos\tilde{\theta} \end{aligned}$$

where $\tilde{\theta} = \tan^{-1}(\gamma_\mathrm{d}^{-1}\tan\theta)$. In the transformed coordinates the expression for the s-wave component in the first order of the perturbation theory in $a_\mathrm{s}/|a_\mathrm{d}|$ reads

$$\Psi_\mathrm{s} = \frac{K_\mathrm{sd}}{a_\mathrm{s}}[\cos 2\alpha(\cos^2\tilde{\theta}\,\nabla_{\tilde{x}}^2 - \nabla_{\tilde{y}}^2) - \sin 2\alpha\cos\tilde{\theta}(\nabla_{\tilde{x}}\nabla_{\tilde{y}} + \nabla_{\tilde{y}}\nabla_{\tilde{x}})]\Psi_\mathrm{d}. \tag{3}$$

Here

$$\Psi_\mathrm{d} = D(r)\exp(\mathrm{i}\varphi)$$

is the conventional vortex solution of the one-component GL theory, ($r = \sqrt{\tilde{x}^2 + \tilde{y}^2}$, $\varphi = \tan^{-1}(\tilde{y}/\tilde{x})$). In order to investigate the structure of Ψ_s we use the following approximation [12] for $D(r)$:

$$D(r) = D_\infty r/\sqrt{r^2 + d^2} \qquad (D_\infty = \sqrt{|a_\mathrm{d}|/b_\mathrm{d}}).$$

Note that

$$d = \sqrt{2}\xi_\mathrm{d} \qquad (\xi_\mathrm{d} = \sqrt{K_\mathrm{d}/|a_\mathrm{d}|})$$

for extreme type-II superconductors. The s-wave OP component distribution in the plane perpendicular to the magnetic field direction has the following form:

$$\Psi_\mathrm{s} = S_3(r)\mathrm{e}^{\mathrm{i}3\varphi} + S_1(r)\mathrm{e}^{\mathrm{i}\varphi} + S_{-1}(r)\mathrm{e}^{-\mathrm{i}\varphi} \tag{4}$$

where

$$\begin{aligned} S_3(r) &= 3D_\infty\frac{K_\mathrm{sd}}{K_\mathrm{d}}\frac{|a_\mathrm{d}|}{a_\mathrm{s}}\frac{r^3}{(r^2+d^2)^{5/2}}[(1+\cos^2\theta)\cos 2\alpha + 2\mathrm{i}\cos\theta\sin 2\alpha] \\ S_1(r) &= -4D_\infty\frac{K_\mathrm{sd}}{K_\mathrm{d}}\frac{|a_\mathrm{d}|}{a_\mathrm{s}}\frac{r(r^2+4d^2)}{(r^2+d^2)^{5/2}}\sin^2\theta\cos 2\alpha \\ S_{-1}(r) &= D_\infty\frac{K_\mathrm{sd}}{K_\mathrm{d}}\frac{|a_\mathrm{d}|}{a_\mathrm{s}}\frac{r(r^2+4d^2)}{(r^2+d^2)^{5/2}}[(1+\cos^2\theta)\cos 2\alpha - 2\mathrm{i}\cos\theta\sin 2\alpha]. \end{aligned} \tag{5}$$

Expressions (4), (5) describe the evolution of the vortex structure as a function of the angles between the external field and the crystalline axes. One can see that the vortex structure exhibits a fourfold symmetry only if $\theta = 0$ (when the magnetic field is parallel to the c-axis); otherwise we have twofold symmetry of the flux line (figure 1). Figure 1 shows the contour plot of the absolute value of the s-wave component in the vortex for $\alpha = 15°$ and different values of θ. Note that this plot is drawn in the rotated (but not rescaled) coordinates (X, Y):

$$\begin{aligned} X &= [x\cos\alpha + y\sin\alpha]\cos\theta - z\sin\theta \\ Y &= -x\sin\alpha + y\cos\alpha \\ Z &= [x\cos\alpha + y\sin\alpha]\sin\theta + z\cos\theta \\ R &= \sqrt{X^2 + Y^2}. \end{aligned} \tag{6}$$

The typical arrangement of s-wave unit vortices in the vicinity of the d-wave core, for different values of θ in the plane perpendicular to the flux line, is shown in figure 2 in the coordinates (6). As the angle between the magnetic field direction and the c-axis increases, two s-wave vortices (A and C) with the same winding number as for the d-wave vortex ($N = +1$) move away from the centre of the flux line. At the same time, the other two s-wave vortices (B and D) move towards the s-wave antivortex (O). When θ approaches a certain critical angle θ^*, the latter two vortices merge with the s-wave antivortex (O) into the central vortex (O′) with a winding number $N = +1$. Finally, when the magnetic field lies in the ab-plane ($\theta = \pi/2$), there is only one s-wave unit vortex with the positive winding number in the d-wave vortex core.

The coordinates of the vortices A and C are

$$\begin{aligned} r_{\mathrm{A,C}} &= d\left[2\frac{\sqrt{(1+\cos^2\theta)^2 + 4\tan^2 2\alpha\cos^2\theta} + 2\sin^2\theta}{\sqrt{(1+\cos^2\theta)^2 + 4\tan^2 2\alpha\cos^2\theta} - \sin^2\theta}\right]^{1/2} \\ \varphi_{\mathrm{A}} &= -\frac{1}{2}\tan^{-1}\left(2\tan 2\alpha\frac{\cos\theta}{1+\cos^2\theta}\right) \qquad \varphi_{\mathrm{C}} = \pi + \varphi_{\mathrm{A}} \end{aligned} \tag{7}$$

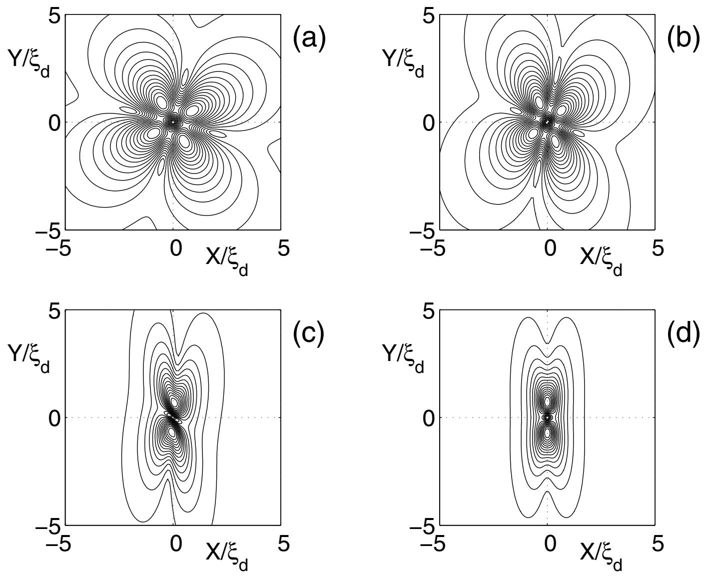

Figure 1. Contour plots of the s-wave OP component for $\alpha = 15°$ and different values of θ: $\theta = 0°$ (a), $\theta = 45°$ (b), $\theta = 80°$ (c), $\theta = 90°$ (d).

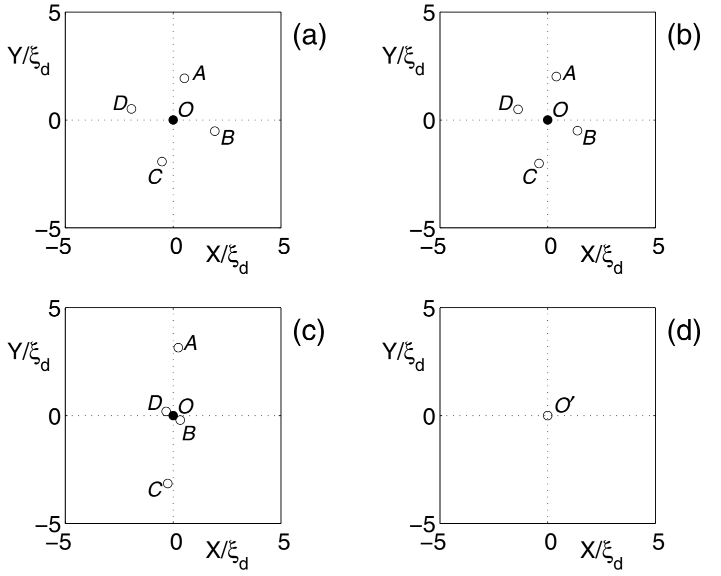

Figure 2. The typical arrangement of s-wave unit vortices in the vicinity of the d-wave vortex for the same angles as in figure 1. Open circles denote s-wave vortices with the same winding number as a d-wave vortex ($N = +1$); black circles denote the s-wave antivortices with $N = -1$.

while the coordinates of the vortices B and D are

$$r_{\mathrm{B,D}} = d\left[2\frac{\sqrt{(1+\cos^2\theta)^2 + 4\tan^2 2\alpha\cos^2\theta} - 2\sin^2\theta}{\sqrt{(1+\cos^2\theta)^2 + 4\tan^2 2\alpha\cos^2\theta} + \sin^2\theta}\right]^{1/2}$$
$$\varphi_{\mathrm{B}} = \frac{\pi}{2} - \frac{1}{2}\tan^{-1}\left(2\tan 2\alpha\frac{\cos\theta}{1+\cos^2\theta}\right) \qquad \varphi_{\mathrm{D}} = \pi + \varphi_{\mathrm{B}}. \tag{8}$$

For θ close to $\pi/2$, the distance between vortices O and A and that between O and C can be approximated by a $1/\cos\theta$ dependence. Note that expressions (7), (8) are written in the coordinate system (3).

The dependences of $r_{\mathrm{A,C}}$ and $r_{\mathrm{B,D}}$ on the angle θ are shown in figure 3 for different values of α. The dependence of θ^* (when the s-wave vortex arrangement changes qualitatively) on the angle α for different anisotropy parameters γ_{d} takes the form (see figure 4)

$$\theta^* = \tan^{-1}\gamma_{\mathrm{d}}\sqrt{\frac{\sqrt{(2\tan^2 2\alpha+5)^2-9} - 2(1-\tan^2 2\alpha)}{5+2\tan^2 2\alpha - \sqrt{(2\tan^2 2\alpha+5)^2-9}}}.$$

Note that the flux line orientation $\alpha = \pi/4$ is a special case, since $S_{-1}(r) \equiv 0$ and the s-wave OP component has a fourfold symmetry for any value of θ. This fact is due to the symmetry of the s + d coupling which cancels the induced s-wave component when the gradients along the crystalline a- and b-axes are equal. The amplitude of the absolute value of Ψ_{s} decreases as $\cos\theta$ with increasing θ and finally becomes zero when θ approaches $\pi/2$.

It is obvious that the internal structure of the flux line affects the dependence of H_{c1} on the magnetic field direction. An additional term of H_{c1}, which depends on α and θ, is proportional to the additional term of the free-energy functional which is due to the presence of a nonzero s-wave component. Within the first order of the perturbation theory, the angle-dependent

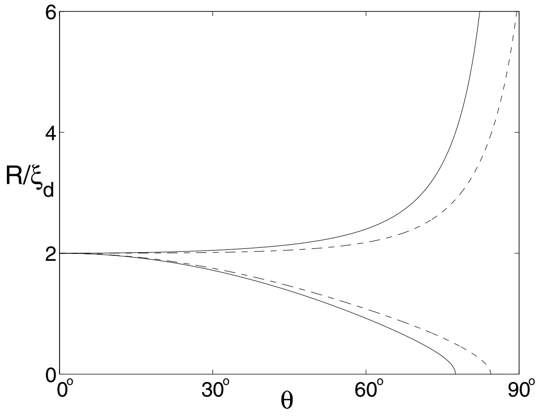

Figure 3. The distance between the d-wave unit vortex and s-wave ones versus the magnetic field direction for $\alpha = 0^\circ$ (solid lines) and $\alpha = 15^\circ$ (dashed lines). The upper lines correspond to the distance between O and A (or C) and the lower lines correspond to the distance between O and B (or D); see figure 2.

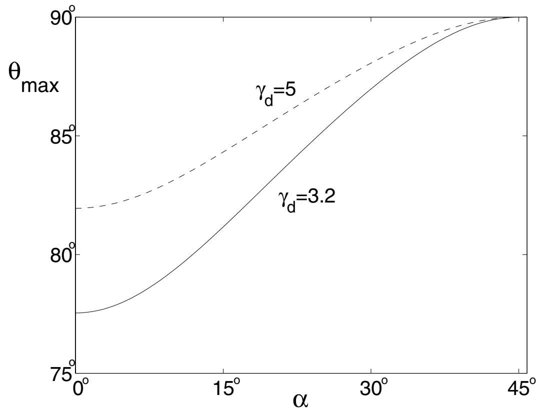

Figure 4. The dependence of the critical angle θ^* on the magnetic field direction in the ab-plane for different values of the anisotropy parameter $\gamma_{\rm d}$.

additional term in the lower critical magnetic field H_{c1} reads

$$\Delta H_{c1}(\theta,\alpha) = \frac{4\pi}{\Phi_0}\,\Delta F(\theta,\alpha) = \frac{4\pi}{\Phi_0} a_{\rm s}\xi_{\rm d}\int_0^{2\pi}{\rm d}\varphi\int_0^{\infty}|\Psi_{\rm s}(r,\varphi)|^2 r\,{\rm d}r$$
$$\cong \frac{\Phi_0}{\lambda_{ab}^2\gamma_{\rm d}}\frac{K_{\rm sd}^2}{K_{\rm d}^2}\frac{|a_{\rm d}|}{a_{\rm s}}\sqrt{\gamma_{\rm d}^2\cos^2\theta+\sin^2\theta}$$
$$\times\left\{\cos^2 2\alpha[(1+\cos^2\tilde\theta)^2+2\sin^4\tilde\theta]+4\sin^2 2\alpha\cos^2\tilde\theta-4\right\}. \qquad (9)$$

This term reflects the presence of the s-wave OP component and possesses a fourfold symmetry in the ab-plane.

3. Tilted vortex structure in quasi-2D superconductors (the strong-anisotropy limit, $s \gg \xi_c$)

In this part we discuss the structure of the s-wave component in the tilted vortex in quasi-2D superconductors. The LD free-energy functional for systems with $({\rm d}_{x^2-y^2}+{\rm s})$-wave pairing has the following form:

$$F = \sum_{n=-\infty}^{+\infty} \int \Big\{ a_\mathrm{d}|\Psi_{\mathrm{d}n}|^2 + a_\mathrm{s}|\Psi_{\mathrm{s}n}|^2 + \frac{b_\mathrm{d}}{2}|\Psi_{\mathrm{d}n}|^4 + \frac{b_\mathrm{s}}{2}|\Psi_{\mathrm{s}n}|^4 + \beta_1|\Psi_{\mathrm{d}n}|^2|\Psi_{\mathrm{s}n}|^2$$
$$+ \frac{\beta_2}{2}(\Psi_{\mathrm{d}n}^2\Psi_{\mathrm{s}n}^{*2} + \Psi_{\mathrm{d}n}^{*2}\Psi_{\mathrm{s}n}^2) + K_\mathrm{s}|\mathbf{\Pi}_n\Psi_{\mathrm{s}n}|^2 + K_\mathrm{d}|\mathbf{\Pi}_n\Psi_{\mathrm{d}n}|^2$$
$$+ K_\mathrm{sd}[(\Pi_{nx}^*\Psi_{\mathrm{s}n}^*\Pi_{nx}\Psi_{\mathrm{d}n} - \Pi_{ny}^*\Psi_{\mathrm{s}n}^*\Pi_{ny}\Psi_{\mathrm{d}n}) + \mathrm{c.c.}]$$
$$+ \frac{\rho_\mathrm{s}}{2}|\Psi_{\mathrm{s}n} - \Psi_{\mathrm{s},n+1}\mathrm{e}^{-\mathrm{i}\chi_{n,n+1}}|^2 + \frac{\rho_\mathrm{d}}{2}|\Psi_{\mathrm{d}n} - \Psi_{\mathrm{d},n+1}\mathrm{e}^{-\mathrm{i}\chi_{n,n+1}}|^2 \Big\}\,\mathrm{d}\boldsymbol{r} \tag{10}$$

where

$$\mathbf{\Pi}_n = \boldsymbol{\nabla} - \mathrm{i}\frac{2\pi}{\Phi_0}\boldsymbol{A}_n$$

and $\boldsymbol{r} = (x, y)$, $\boldsymbol{\nabla} = \partial/\partial\boldsymbol{r}$, $\boldsymbol{A}_n = (A_x(\boldsymbol{r}, z = ns), A_y(\boldsymbol{r}, z = ns))$, $\boldsymbol{H} = \mathrm{curl}\,\boldsymbol{A}$, $\Psi_{\mathrm{d}n}(\boldsymbol{r})$ and $\Psi_{\mathrm{s}n}(\boldsymbol{r})$ are the OP components in the nth layer ($z = ns$), s is the interlayer spacing, and

$$\chi_{n,n+1} = \frac{2\pi}{\Phi_0}\int_{ns}^{(n+1)s} A_z\,\mathrm{d}z.$$

The equations for Ψ_d and Ψ_s are written as follows:

$$a_\mathrm{d}\Psi_{\mathrm{d}n} + b_\mathrm{d}|\Psi_{\mathrm{d}n}|^2\Psi_{\mathrm{d}n} + \beta_1|\Psi_{\mathrm{s}n}|^2\Psi_{\mathrm{d}n} + \beta_2\Psi_{\mathrm{s}n}^2\Psi_{\mathrm{d}n}^* - K_\mathrm{d}\mathbf{\Pi}_n^2\Psi_{\mathrm{d}n}$$
$$+ K_\mathrm{sd}(\Pi_{ny}^2 - \Pi_{nx}^2)\Psi_{\mathrm{s}n} + \frac{\rho_\mathrm{d}}{2}(2\Psi_{\mathrm{d}n} - \Psi_{\mathrm{d},n+1}\mathrm{e}^{-\mathrm{i}\chi_{n,n+1}} - \Psi_{\mathrm{d},n-1}\mathrm{e}^{\mathrm{i}\chi_{n-1,n}}) = 0$$
$$a_\mathrm{s}\Psi_{\mathrm{s}n} + b_\mathrm{s}|\Psi_{\mathrm{s}n}|^2\Psi_{\mathrm{s}n} + \beta_1|\Psi_{\mathrm{d}n}|^2\Psi_{\mathrm{s}n} + \beta_2\Psi_{\mathrm{d}n}^2\Psi_{\mathrm{s}n}^* - K_\mathrm{s}\mathbf{\Pi}_n{}^2\Psi_{\mathrm{s}n}$$
$$+ K_\mathrm{sd}(\Pi_{ny}^2 - \Pi_{nx}^2)\Psi_{\mathrm{d}n} + \frac{\rho_\mathrm{s}}{2}(2\Psi_{\mathrm{s}n} - \Psi_{\mathrm{s},n+1}\mathrm{e}^{-\mathrm{i}\chi_{n,n+1}} - \Psi_{\mathrm{s},n-1}\mathrm{e}^{\mathrm{i}\chi_{n-1,n}}) = 0. \tag{11}$$

In this section we restrict ourselves to the most interesting case of the angle domain $\lambda_J/s \ll \tan\theta \ll \lambda_c/s$ ($\lambda_J = \sqrt{2K_\mathrm{d}/\rho_\mathrm{d}}$ is the Josephson length), since in this angle domain the difference between the quasi-2D theory and the 3D theory is at its most significant. The induced s-wave component appears to be essential only in the $\boldsymbol{r}$-domains close to the vortex line axis. As a result, for angles $\tan\theta \ll \lambda_c/s$ (λ_c is the magnetic field penetration depth [5]) we can neglect the screening effects in these domains and put $\boldsymbol{A} = 0$. We again rotate the coordinate system so that the vortex line lies in the xz-plane. The expression for the s-wave component in the first-order perturbation theory (see section 2) in the new coordinates reads

$$\Psi_{\mathrm{s}n} = \frac{K_\mathrm{sd}}{a_\mathrm{s}}[(\nabla_x^2 - \nabla_y^2)\Psi_{\mathrm{d}n}\cos 2\alpha - (\nabla_x\nabla_y + \nabla_y\nabla_x)\Psi_{\mathrm{d}n}\sin 2\alpha]. \tag{12}$$

Outside the normal core regions we can put $\Psi_{\mathrm{d}n} = \exp(\mathrm{i}\varphi_n)$, where φ_n is the phase in the nth layer. The phase φ_n can be obtained from the equation for the d-wave OP, where the s-wave OP is neglected. Thus, φ_n is the solution for the tilted vortex in the conventional layered superconductors (see e.g. reference [5]).

In the layered compounds the flux line is a stack of pancakes connected by Josephson strings. There are several regions [5] in each layer with qualitatively different behaviours of the phase φ_n (see figure 5). The normal core region ($r < \xi_{ab}$) is an area of suppression of the absolute value of the OP. The 2D core region is an area where $|\Psi_{\mathrm{d}n}| = \text{constant}$, while the phase is that of a pancake. Inside this 2D London region ($r < \lambda_J$) the phase is given by $\varphi_n = \tan^{-1}(x/y)$. In a Josephson string ($\lambda_J \ll |x| \ll s\tan\theta$ and $|y| < \lambda_J$) we have $|\nabla_x\varphi_n| \ll |\nabla_y\varphi_n|$; therefore, we neglect $\nabla_x\varphi_n$ and use the linear approximation for the phase:

$$\nabla_y\varphi_n(x, y) = \frac{\varphi_n(x, \lambda_J) - \varphi_n(x, -\lambda_J)}{2\lambda_J} = \frac{\pi}{2\lambda_J}.$$

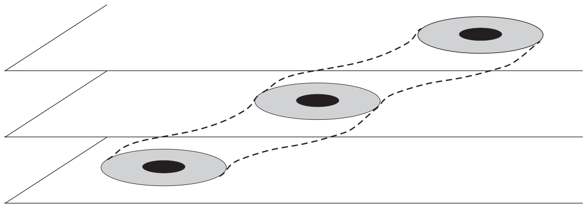

Figure 5. Normal cores (black regions), 2D cores (grey regions), and the Josephson string (the regions between dashed lines) are shown schematically for a tilted vortex in the angular domain $\lambda_J/s \ll \tan\theta \ll \lambda_c/s$.

For a 3D London region ($|x| \gg s\tan\theta$ and $|y| \gg \lambda_J$) we have a conventional anisotropic GL solution for the phase:

$$\varphi_n = \tan^{-1}\left(\frac{\gamma}{\tan\theta}\frac{x}{y}\right)$$

where $\gamma = \lambda_J/s$ is the anisotropy parameter.

The additional term in the functional F, which is connected with the s-wave OP, reads

$$\begin{aligned}\Delta F(\theta,\alpha) = &-\frac{\Phi_0^2 s\xi_{\mathrm{d}}^2}{32\pi^3\lambda_{ab}^2}\frac{K_{\mathrm{sd}}^2}{K_{\mathrm{d}}^2}\frac{|a_{\mathrm{d}}|}{a_{\mathrm{s}}}\\ &\times\sum_{n=-\infty}^{+\infty}\int\{([(\nabla_x\varphi_n)^2-(\nabla_y\varphi_n)^2]\cos 2\alpha - 2\nabla_x\nabla_y\varphi_n\sin 2\alpha)^2\\ &+([\nabla_y^2-\nabla_x^2]\varphi_n\cos 2\alpha - [\nabla_x\nabla_y+\nabla_y\nabla_x]\varphi_n\sin 2\alpha)^2\}\,\mathrm{d}\boldsymbol{r}\end{aligned} \tag{13}$$

where $\xi_{\mathrm{d}} = \sqrt{K_{\mathrm{d}}/|a_{\mathrm{d}}|}$ is the coherence length of the d-wave component in the ab-plane.

Within our approach we can calculate the vortex energy per unit length in the most interesting interval of angles θ, where $\lambda_J/s \ll \tan\theta$. We choose this interval of angles because the main contribution to the integral in (13) comes from the Josephson string and for other angles the α-dependent additional term in H_{c1} is neglected.

The α-dependent correction to H_{c1} (which reflects the existence of the s-wave OP component) is given by the expression

$$\Delta H_{c1}(\theta,\alpha) = \frac{4\pi\cos\theta}{\Phi_0 sN}\{\Delta F(\theta,\alpha) - \Delta F(\theta=0,\alpha)\} \simeq \frac{\pi\Phi_0\xi_{\mathrm{d}}^2 s}{\lambda_{ab}^2\lambda_J^3}\frac{K_{\mathrm{sd}}^2}{K_{\mathrm{d}}^2}\frac{|a_{\mathrm{d}}|}{a_{\mathrm{s}}}\sin\theta\cos^2 2\alpha \tag{14}$$

where N is the number of layers.

4. Summary and discussion

To summarize, we have studied the core structure of the tilted vortices in ($\mathrm{d}_{x^2+y^2}$ + s)-wave superconductors and its continuous evolution as the field direction changes. Except when the magnetic field is close to the ab-plane, the s-wave component of the order parameter has an antivortex at the central core and four satellite vortices. With increasing angle between the magnetic field direction and the c-axis (θ), two s-wave vortices move away from the centre of the flux line, while the other two s-wave vortices move towards the core. As θ approaches the critical value θ^*, these two latter s-wave vortices merge with the central s-wave antivortex and form one vortex with a positive winding number. When the magnetic field lies in the ab-plane, the s-wave component has one vortex at the centre of the flux line, and the amplitude of the s-wave component is strongly dependent on the angle α and vanishes for $\alpha = \pi/4$ due to the symmetry of the functional. For the angles $\theta < \theta^*$ the distribution of the s-wave OP component is similar to the one obtained by numerical calculations [9]. In contrast, for the angular interval $\theta > \theta^*$ the behaviour of our solution differs qualitatively from the distribution obtained on the basis of the numerical analysis [9]. Instead of the five s-wave vortices, we have only three s-wave vortices. In the strong-anisotropy limit ($s \gg \xi_s$) we studied the structure of the s-wave component in the most interesting interval of angles $\lambda_J/s \ll \tan\theta \ll \lambda_c/s$ where the deviations from the 3D theory are strongest. For both anisotropy limits we found an angle-dependent correction to the lower critical field of a (d + s)-wave superconductor, which reflects the presence of the s-wave OP and possesses a fourfold symmetry in the ab-plane. This additional term in H_{c1} increases with decrease of temperature and is likely to be essential for T close to T_{cs}.

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